

TRANSPORT PHENOMENA IN FREE TURBULENT FLOWS

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Abstract—An analysis is made to find the transport properties of a free turbulent flow using an entity model previously shown to have good prediction capability for pipe flow. It is shown that the double structure of the turbulent eddies and the anisotropic shape of the larger eddies can be satisfactorily incorporated into the model without undue complexity. The analysis results in expressions for the various diffusivities in terms of properties of the turbulence. Predictions can be made for the diffusivity ratios and their dependence on molecular properties of the fluid determined. The results are shown to be in good agreement with the available experimental data.

NOMENCLATURE

D ,	molecular mass diffusion coefficient;
N_{R_0} ,	initial entity Reynolds number;
Pr ,	molecular Prandtl number;
Pr_T ,	turbulent Prandtl number;
Sc ,	molecular Schmidt number;
Sc_T ,	turbulent Schmidt number;
(X, Y, Z) ,	co-ordinate axes;
α ,	molecular thermal diffusion coefficient;
ε ,	diffusion coefficient due to entity migration;
ε_H ,	coefficient for thermal energy diffusion;
ε_m ,	coefficient for mass diffusion;
ε_μ ,	coefficient for momentum diffusion;
ρ ,	fluid density;
μ ,	molecular viscosity;
ψ_x, ψ_y, ψ_z ,	shape distortion factors for momentum interaction;
ψ^1 ,	shape distortion factor for heat and mass transfer.

Superscript

+

used when transport between large entities is dominated by small entity migration.

1. INTRODUCTION

IT HAS been recognized for some time that the Reynolds relationship inadequately describes the connection between the transport of momentum and of energy or mass in a turbulent fluid. If valid, the relation implies equality between the turbulent diffusivities of momentum and thermal energy, ε_μ and ε_H . Experimental values of the ratio ($\varepsilon_H/\varepsilon_\mu$), however, have been found which lie in the range 0.0-1.4 for pipe flow and 1.0-2.4 for a free turbulent flow. In addition it has been shown in pipe flow that ($\varepsilon_H/\varepsilon_\mu$) depends not only upon the structure of the turbulent flow but also upon the properties of the substance as indicated by the Prandtl number of the fluid, or in other words upon the ratio between the *molecular* diffusivities. Little attention appears to have been given to the possibility of a similar dependence for free

turbulent flows despite a certain amount of experimental evidence to its effect.

In the case of pipe flow attempts have been made [1-5] to analyse the transport mechanism and thus predict values for the ratio $(\epsilon_H/\epsilon_\mu)$. A brief summary of the results of these previous theories is presented in a recent paper by Tyldesley and Silver [6]. In the field of free turbulence several models have been proposed to represent the structure of the flow and perhaps the earliest and best known of these is Taylor's vorticity transport theory [7] which produces a unique value for the ratio $(\epsilon_H/\epsilon_\mu)$ of 2.0. This does not accord with much of the experimental data, and furthermore the structure of the flow implied by Taylor's model conflicts with observations of Townsend [8] particularly with regard to the direction of the vorticity axes. More recent models have assumed the presence of large scale inhomogeneities within the flow which are convected between the internal turbulent fluid and the external quiescent flow. A limiting case of this type of structure is when the inhomogeneities retain the properties of the region of creation without any molecular diffusion across their surfaces and this has been discussed by several workers including Corrsin [9], Batchelor [10], Obukhoff [11], Herlin and Herrmann [12], Lin and Hayes [13]. A modification of this approach in which molecular diffusion is allowed during the motion has been made by Proudian and Feldman [14] using a mixing lag model. In none of these theories, however, is it clear what the interrelation is between transport in turbulent pipe flow and free turbulence.

In this present paper it is shown how the analysis previously given by Tyldesley and Silver [6] for the transport properties of a turbulent pipe flow may be applied to a free turbulent flow. The theory predicts values for $(\epsilon_H/\epsilon_\mu)$ which are in good agreement with available experimental results and furthermore enables the effect of molecular Prandtl number and Schmidt number to be assessed where experimental evidence is lacking. By nature of

its derivation the theory is completely compatible with heat and mass transport data in turbulent pipe flow.

2. THE DIFFUSIVITY RATIOS $(\epsilon_H/\epsilon_\mu)$ AND $(\epsilon_m/\epsilon_\mu)$ IN FREE TURBULENCE

In free turbulent flows, such as occur in the mixing region of a jet or in the wake behind a blunt object, it has been shown by Townsend [15] and many other workers that the transport between the inner and outer regions of the turbulent flow is carried out by large entities of a scale comparable with the flow dimensions. At the same time there exists an entity system of a much smaller scale which can cause transport between the large entities. Furthermore the large entity system is known to be anisotropic† while the small scale system is unlikely to be so. It is therefore apparent that in free turbulent flows the large scale entity system cannot be described adequately by a scale parameter whose expected value is independent of direction. The theory previously given by Tyldesley and Silver [6], however, correctly describes this situation if in the analysis the distortion factor ψ is given different values ψ_x, ψ_y, ψ_z , for the motion of the entity in each of the three co-ordinate directions.

Insertion of these values into equation (10) of Tyldesley and Silver [6] yields the result that

$$\epsilon_\mu = \frac{2\mu}{27} \left\langle \psi_y N_R^2 \right\rangle \left\langle \frac{1}{(4 - \psi_y/\psi_x)} \right\rangle \quad (1)$$

$$\epsilon_H = \frac{\mu}{9} \left\langle \psi^1 N_R^2 \right\rangle \left\langle \frac{Pr}{[1 + 3Pr(\psi^1/\psi_y)]} \right\rangle \quad (2)$$

$$\epsilon_m = \frac{\mu}{9} \left\langle \psi^1 N_R^2 \right\rangle \left\langle \frac{Sc}{[1 + 3Sc(\psi^1/\psi_y)]} \right\rangle \quad (3)$$

which reduce to the previous expressions for ϵ_μ , ϵ_H and ϵ_m if $\psi_x = \psi_y$.

If now an analysis is made for the transport across the flow field it is clear that this will be described exactly by equations (1) (2) and (3)

† In this paper "anisotropic" refers in particular to the scale of the fluid entities.

with the exception that the transport *between* the large entities will be influenced by the small entity system and hence the molecular properties μ , Pr and Sc must be replaced by the combined molecular and small entity system properties. Consequently the transport coefficients for the large entity system ε_μ^+ , ε_H^+ , ε_m^+ are given by

$$\varepsilon_\mu^+ = \frac{2(\varepsilon_\mu + \mu)}{27} \left\langle \frac{\psi_y^+ N_R^{+2}}{(4 - \psi_y^+/\psi_x^+)} \right\rangle \quad (4)$$

$$\varepsilon_H^+ = \frac{(\varepsilon_\mu + \mu)}{9} \left\langle \frac{\psi^{1+} N_R^{+2} Pr_T}{[1 + 3Pr_T(\psi^{1+}/\psi_y^+)]} \right\rangle \quad (5)$$

$$\varepsilon_m^+ = \frac{(\varepsilon_\mu + \mu)}{9} \left\langle \frac{\psi^{1+} N_R^{+2} Sc_T}{[1 + 3Sc_T(\psi^{1+}/\psi_y^+)]} \right\rangle \quad (6)$$

where $Pr_T = (\varepsilon_\mu + \mu)/(\varepsilon_H + \alpha)$, $Sc_T = (\varepsilon_\mu + \mu)/(\varepsilon_m + D)$ and ε_μ , ε_H , ε_m refer to the small scale entity system. The new distortion factors ψ_x^+ , ψ_y^+ , ψ^{1+} and Reynolds Number N_R^+ refer to the large scale entity system.

Assuming that the small scale entity system is isotropic then ε_μ , ε_H and ε_m are as given by equations (1), (2) and (3) with $\psi_x = \psi_y = \psi_z$ and the substitution in equations (4), (5) and (6) of these relationships gives

$$\varepsilon_\mu^+ = \frac{2(\varepsilon_\mu + \mu)}{27} \left\langle \frac{\psi_y^+ N_R^{+2}}{(4 - \psi_y^+/\psi_x^+)} \right\rangle \quad (7)$$

$$\varepsilon_H^+ = \frac{(\varepsilon_\mu + \mu)}{27} \left\langle \frac{Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)\psi^{1+} N_R^{+2}}{3Pr^2\langle\psi_y N_R^2\rangle + 27(1 + 3Pr) + Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)} \right\rangle \quad (8)$$

$$\varepsilon_m^+ = \frac{(\varepsilon_\mu + \mu)}{27} \left\langle \frac{Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)\psi^{1+} N_R^{+2}}{3Sc^2\langle\psi_y N_R^2\rangle + 27(1 + 3Sc) + Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)} \right\rangle \quad (9)$$

Consequently the diffusivity ratios and are given by

$$\frac{\varepsilon_H^+}{\varepsilon_\mu^+} = \left\langle \frac{Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)(4 - \psi_y^+/\psi_x^+)}{2[3Pr^2\langle\psi_y N_R^2\rangle + 27(1 + 3Pr) + Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)]} \right\rangle \quad (10)$$

$$\frac{\varepsilon_m^+}{\varepsilon_\mu^+} = \left\langle \frac{Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)(4 - \psi_y^+/\psi_x^+)}{2[3Sc^2\langle\psi_y N_R^2\rangle + 27(1 + 3Sc) + Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)(\psi^{1+}/\psi_y^+)]} \right\rangle \quad (11)$$

In pipe flow there is no evidence of any significant anisotropic scale features except perhaps in the neighbourhood of the walls and

consequently for this type of flow it may be assumed that $(\psi^{1+}/\psi_y^+) \doteq 1.0$ and $(\psi_y^+/\psi_x^+) \doteq 1.0$. Inserting these values into equations (10) and (11) gives the result shown in Fig. 1. The extreme curves for $\langle\psi N_R^2\rangle = 0$ and $\langle\psi N_R^2\rangle = \infty$ represented by the equations

$$\frac{\varepsilon_H}{\varepsilon_\mu} = \left(\frac{9Pr}{2 + 6Pr} \right) \quad (12)$$

$$\frac{\varepsilon_m}{\varepsilon_\mu} = \left(\frac{3 + 9Pr}{2 + 9Pr} \right) \quad (13)$$

have been shown previously by Tyldesley and Silver [6] to satisfactorily predict the extreme values of experimental data and also the direction of variation with increasing Reynolds number. At extremely low values for Pr or Sc , however, the entity Péclet number (PrN_R) becomes most important in determining the value of $(\varepsilon_H/\varepsilon_\mu)$ and the extreme values represented by equations (12) and (13) may be less readily achieved in experimental flows.

It is clear from equations (10) and (11) that in the limit as Pr or $Sc \rightarrow 0$ then $(\varepsilon_H^+/\varepsilon_\mu^+)$ and $(\varepsilon_m^+/\varepsilon_\mu^+) \rightarrow 0$.

In free turbulence this situation is unlikely to describe the large entity system transport since here the flow structure has been shown by Grant [16], Keffer [17] and others to be

dominated by large anisotropic entities moving transverse to the main flow direction. The form of these large scale entities is discussed by

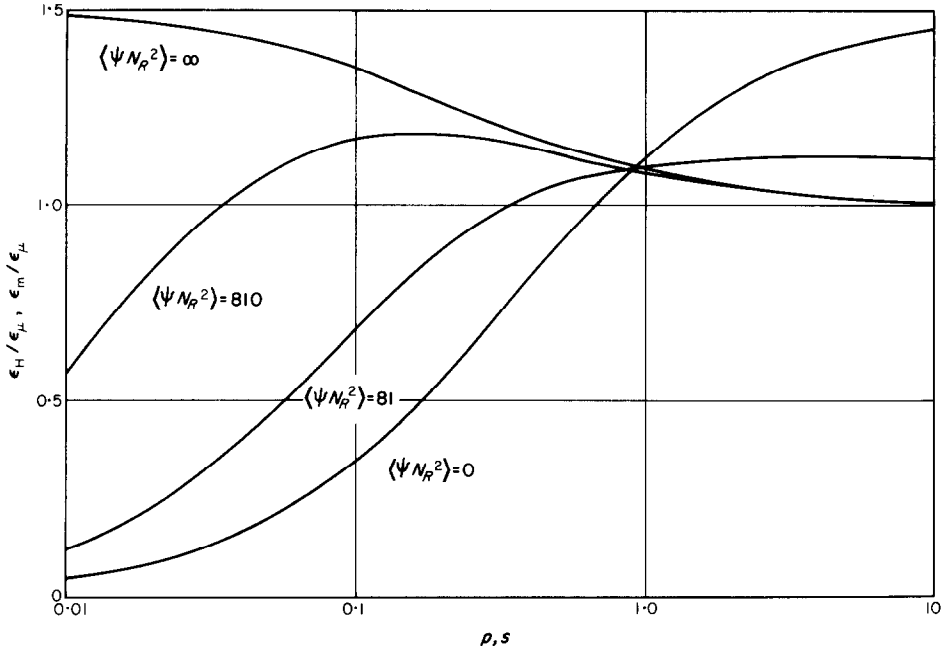


FIG. 1. The effect of molecular Prandtl and Schmidt number on the ratios of turbulent diffusivities in turbulent pipe flow.

Townsend [8] and he shows that they are elongated in the direction of the main flow. Experimental data given by Townsend shows that the longitudinal turbulent scale is greater than the lateral scale by a factor of 7.0 or more. Tyldesley and Silver have shown previously [6] that the turbulent scale is proportional to the entity scale and thus in order to conform with the observations the large entity system must be composed of entities which, on the average, have a scale in direction of the mean flow much greater than that in the lateral direction.

An estimate of the value of the ratio (ψ_y^+/ψ_x^+) can be made by considering the possibility that the large entity may be approximated by a spheroid for which values of ψ are available for various direction of motion. Happel and Brenner [18] give results which show that for a prolate spheroid, a needle-line object, (ψ_y^+/ψ_x^+) has a minimum value of 0.5 while for an oblate spheroid or disc shape the corresponding limit is 0.66.

The decrease in (ψ_y^+/ψ_x^+) as the diameter ratio is increased is shown to be rapid. Corresponding values for the ratio $((\psi^{1+}/\psi_y^+))$ can be obtained since analyses have been made for the heat transfer to spheroids (c.f. Carslaw and Jaeger [19], Levine [20], Weber [21] for equations and their solutions.) The results show that the limiting values for oblate and prolate spheroids are 1.35 and 1.33 respectively, in this case the limits being achieved rather more slowly as the diameter ratio is varied. From this information it seems reasonable to assume that the extreme values of 0.5 and 1.33 for the prolate spheroid are possibly not achieved and a conservative estimate might be that $(\psi_y^+/\psi_x^+) \doteq 0.69$ and $(\psi^{1+}/\psi_y^+) \doteq 1.27$ which corresponds to a prolate spheroid having a length/diameter ratio of 10, which is of the same order as found by experiment.

Substituting these values in equations (10) and (11) yields the following result for the diffusivity ratios.

$$\frac{\epsilon_H^+}{\epsilon_\mu^+} = \left\langle \frac{2.1 Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)}{3Pr^2\langle\psi_y N_R^2\rangle + 27(1 + 3Pr) + 1.27Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)} \right\rangle \quad (14)$$

$$\frac{\epsilon_m^+}{\epsilon_\mu^+} = \left\langle \frac{2.1 Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)}{3Sc^2\langle\psi_y N_R^2\rangle + 27(1 + 3Sc) + 1.27Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)} \right\rangle \quad (15)$$

If now the extreme values, 0.5 and 1.33 are substituted in equations (10) and (11) then the result is that

$$\frac{\epsilon_H^+}{\epsilon_\mu^+} = \left\langle \frac{2.33Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)}{3Pr^2\langle\psi_y N_R^2\rangle + 27(1 + 3Pr) + 1.33Pr(1 + 3Pr)(2\langle\psi_y N_R^2\rangle + 81)} \right\rangle \quad (16)$$

$$\frac{\epsilon_m^+}{\epsilon_\mu^+} = \left\langle \frac{2.33Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)}{3Sc^2\langle\psi_y N_R^2\rangle + 27(1 + 3Sc) + 1.33Sc(1 + 3Sc)(2\langle\psi_y N_R^2\rangle + 81)} \right\rangle \quad (17)$$

These results are shown plotted in Fig. 2 together with experimental values obtained in axisymmetric free turbulent flows. It can be seen from these results that the range of experimental points lies between the predicted curves indicating that the actual entity distortion lies between the two extremes considered in the analysis and possibly depends upon details of the flow configuration. The trend of relatively

lower values of $(\epsilon_H^+/\epsilon_\mu^+)$ for water as compared to gas data is predicted correctly and can be seen from equations (14) and (15) to be the direct result of variations in molecular Prandtl number for the two fluids.

3. THE DIFFUSIVITY RATIOS $(\epsilon_H/\epsilon_\mu)$ AND $(\epsilon_m/\epsilon_\mu)$ IN INTERMITTENT TURBULENT FLOWS

In the wake flow behind a cylinder or in the

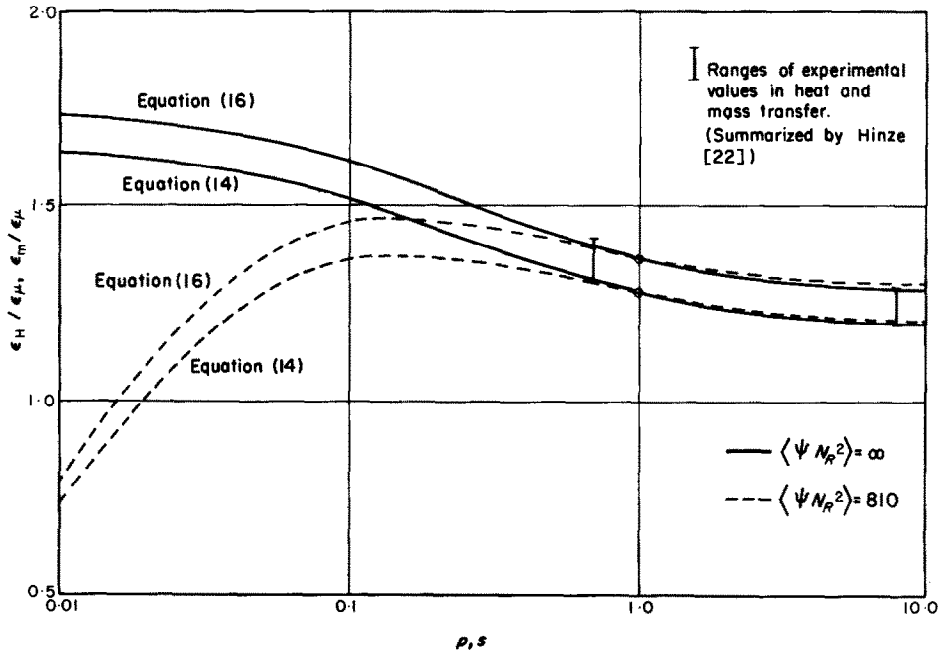


FIG. 2. The effect of molecular Prandtl and Schmidt number on the ratios of turbulent diffusivities in free turbulent flow.

mixing region of a turbulent jet the flow is found to be partially turbulent and partially laminar in character. The characteristic features of such a flow are that the boundary between the laminar and turbulent fluid is distorted by billows of turbulent fluid which randomly emerge from the turbulent core but never become detached from the core. Isolated regions of laminar or turbulent fluid are not generally found. Experimental data indicates that the mean velocity of the turbulent billows is not significantly different from the adjacent laminar fluid but if there is a significant difference in either temperature or chemical composition between the turbulent core and the external laminar flow then there is a similar difference of comparable magnitude between the turbulent billows and the adjacent laminar fluid. Townsend [8] has shown that this difference arises because the turbulent billows are constrained by the fluctuating pressure gradients rather than by shear forces and since in the transport of heat and mass there is no equivalent additional constraint their transport is relatively unimpeded.

This situation can be analysed using equation (4) if it is realised that the effect on the motion of an entity of this additional constraint due to fluctuating pressure gradients is similar to the effect of increasing the momentum diffusivity. If therefore ε_μ in equation (4) is replaced by

$\gamma\varepsilon_\mu$ where $\gamma > 1.0$ then the analysis can proceed as before. As a result the relationship for the ratio can be written by analogy with the development of equation (10).

$$\frac{\varepsilon_H^+}{\varepsilon_\mu^+} \doteq \left\langle \frac{3\gamma Pr_T(\psi^{1+}/\psi_y^+)(4 - \psi_y^+/\psi_x^+)}{2(1 + 3\gamma Pr_T(\psi^{1+}/\psi_y^+))} \right\rangle \quad (18)$$

which when γ becomes large may be approximated by

$$\frac{\varepsilon_H^+}{\varepsilon_\mu^+} \doteq \left\langle \frac{4 - \psi_y^+/\psi_x^+}{2} \right\rangle \quad \gamma \gg 1.0. \quad (19)$$

Since the turbulent billows are the result of the outward motion of the large scale entities in the core of the turbulent flow they will have similar scale distortions and hence $(\psi_y^+/\psi_x^+) < 1.0$ and values are likely to lie in the range 0.69–0.5. The corresponding values of $(\varepsilon_H^+/\varepsilon_\mu^+)$ lie in the range 1.66–1.75 and are independent of any molecular properties of the fluids.

In Table 1 a brief summary of experimental results in plane jets or wake flows is given together with alternative theoretical predictions.

It can be seen from Table 1 that the present predictions are in reasonable agreement with the range of values obtained by experiment.

CONCLUSIONS

The analysis given in this paper shows application of the entity model to the problem of transport in a free turbulent flow. The pre-

Table 1. Experimental results and alternative theories

Flow field	$\left(\frac{\varepsilon_H}{\varepsilon_\mu}\right)$	$\left(\frac{\varepsilon_m}{\varepsilon_\mu}\right)$	Author
Planar wake	1.85	—	Fage and Faulkner
Plane jet	1.85	—	Reichardt
Plane jet	1.7–2.4	—	Van der Hegge Zynen
Theory	$\left(\frac{\varepsilon_H}{\varepsilon_\mu}\right)$	$\left(\frac{\varepsilon_m}{\varepsilon_\mu}\right)$	Author
Vorticity transport	2.0	—	Taylor
Inductive theory	2.0	—	Reichardt
Entity model	1.66–1.75	1.66–1.75	Present

dictions of diffusivity ratios are in good agreement with experimental results and have been obtained using relatively simple assumptions for the structure of the turbulence and without the use of any detailed experimental data. Whilst it is beyond dispute that the turbulent structure is more complex than assumed in the analysis, the results of the analysis for both pipe flow and free turbulence show that the pertinent details of the flow are deducible from the model. A theory which can be used successfully in both types of turbulent flow is much to be desired and the present theory achieves this purpose despite its simplicity.

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Résumé—Une analyse est faite pour trouver les propriétés de transport d'un écoulement turbulent libre employant un modèle d'entité que l'on a montré auparavant avoir une bonne aptitude pour prédire l'écoulement dans un tuyau. On montre que la structure double des tourbillons turbulents et la forme anisotrope des tourbillons les plus importants peut être incorporée d'une façon satisfaisante dans le modèle sans complexité excessive. L'analyse aboutit à des expressions pour les différentes diffusivités en fonction des propriétés de la turbulence. On peut prédire les rapports des diffusivités et déterminer leurs dépendances des propriétés moléculaires du fluide. On montre que les résultats sont en bon accord avec les résultats expérimentaux disponibles.

Zusammenfassung—Mit Hilfe eines Grössenmodells das kürzlich zu guten Ergebnissen für die Rohrströmung führte, wurde eine Analyse durchgeführt um die Transporteigenschaften einer freien turbulenten Strömung zu ermitteln. Es wird gezeigt, dass die Doppelstruktur der turbulenten Wirbel und die anisotrope Form der grösseren Wirbel zufriedenstellend in das Modell eingebaut werden kann ohne übermässige Komplexität zu verursachen. Die Analyse liefert Ausdrücke für die verschiedenen Austauschgrössen in Form von Turbulenzeigenschaften. Austauschverhältnisse und ihre Abhängigkeit von Molekulareigenschaften der Flüssigkeit können berechnet werden. Die Ergebnisse stehen in guter Übereinstimmung mit verfügbaren Versuchswerten.

Аннотация—Свойства переноса свободного турбулентного потока выявляются с помощью известной модели, на которой ранее было успешно рассчитано течение в трубе. Показано, что двойную структуру турбулентных вихрей и анизотропную форму больших вихрей можно легко воспроизвести на этой модели. В результате анализа получены выражения для различных значений коэффициента диффузии, выраженных через свойства турбулентности. Можно рассчитать отношения коэффициентов диффузии и их зависимость от молекулярных свойств жидкости. Показано, что результаты хорошо согласуются с имеющимися экспериментальными данными.